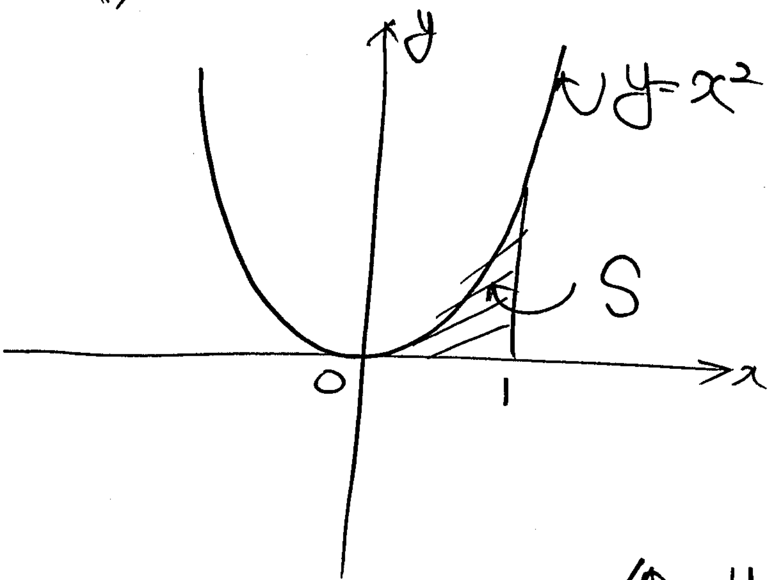
10. 정적분의 활용

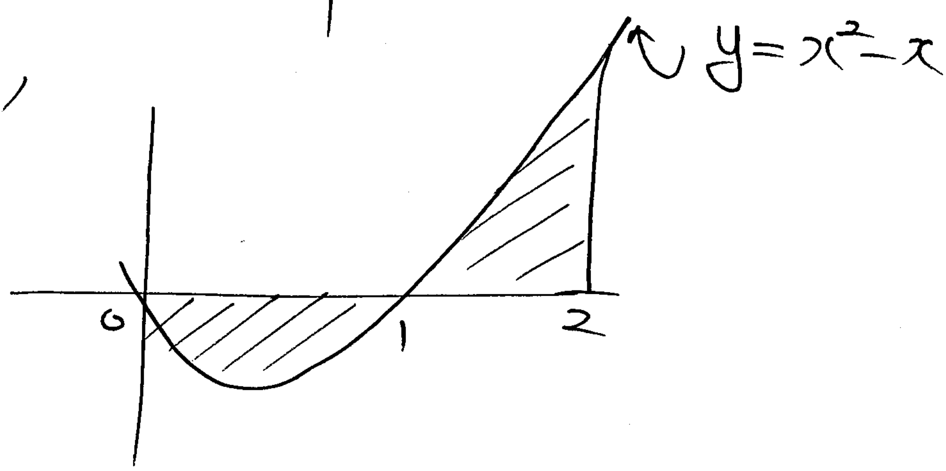
P310

EX) (1)



$$S=\int_0^1 x^2\,dx$$
$$=\left[\frac{x^3}{3}\right]_0^1=\frac{1}{3}$$

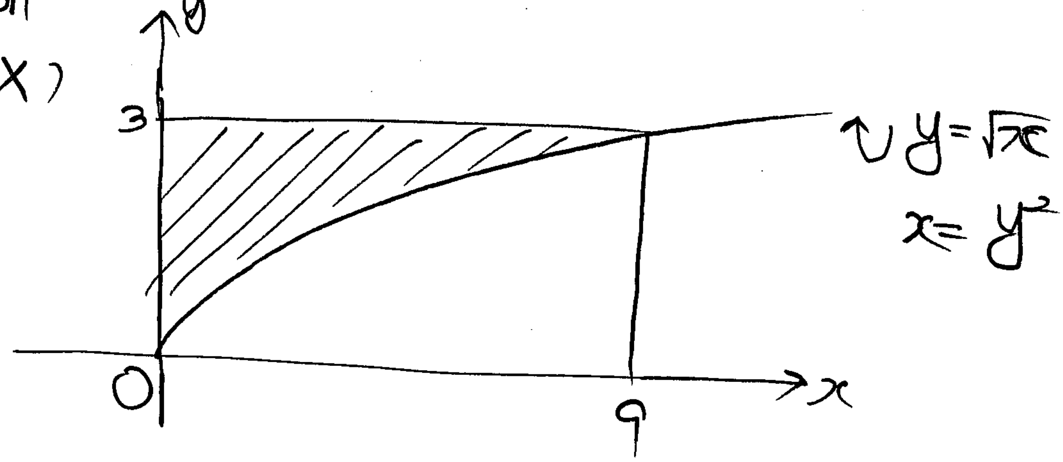
(2)



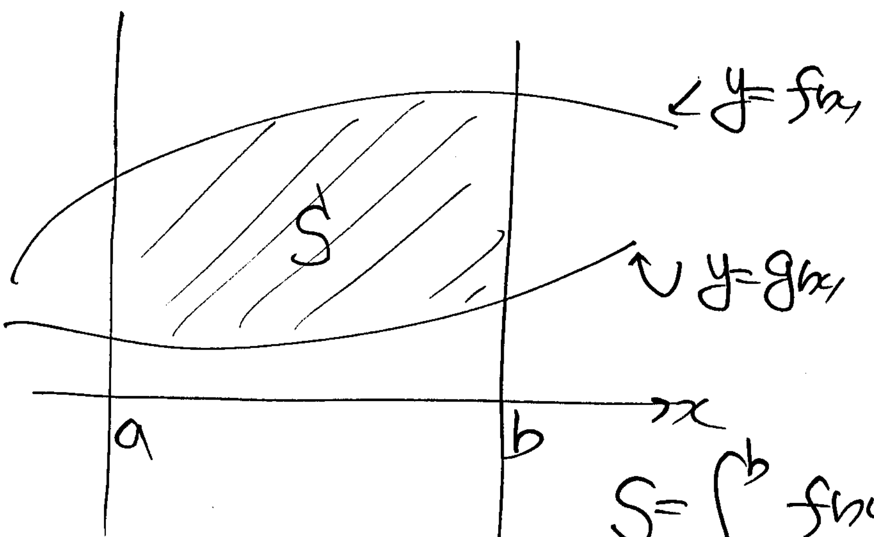
$$S=\int_0^1(-x^2+x)\,dx+\int_1^2(x^2-x)\,dx$$
$$=\left[-\frac{x^3}{3}+\frac{x^2}{2}\right]_0^1+\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_1^2$$
$$=\left(-\frac{1}{3}+\frac{1}{2}\right)+\frac{1}{3}(8-1)-\frac{1}{2}(4-1)$$
$$=-\frac{1}{3}+\frac{1}{2}+\frac{7}{3}-\frac{3}{2}=1$$

P311

EX)

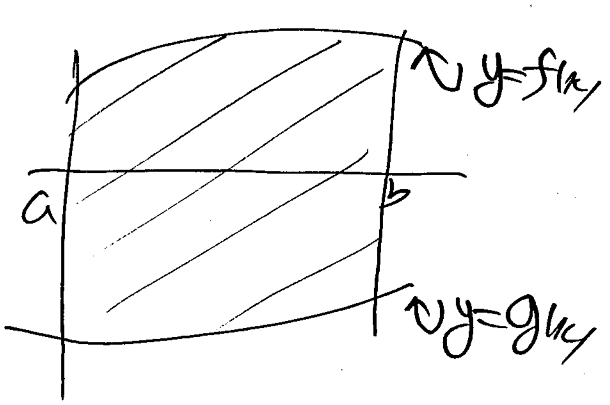


$$S = \int_0^3 y^2\,dy = \left[\frac{1}{3}y^3\right]_0^3 = 9$$



$$S = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$= \boxed{\int_a^b (f(x) - g(x))\,dx}$$

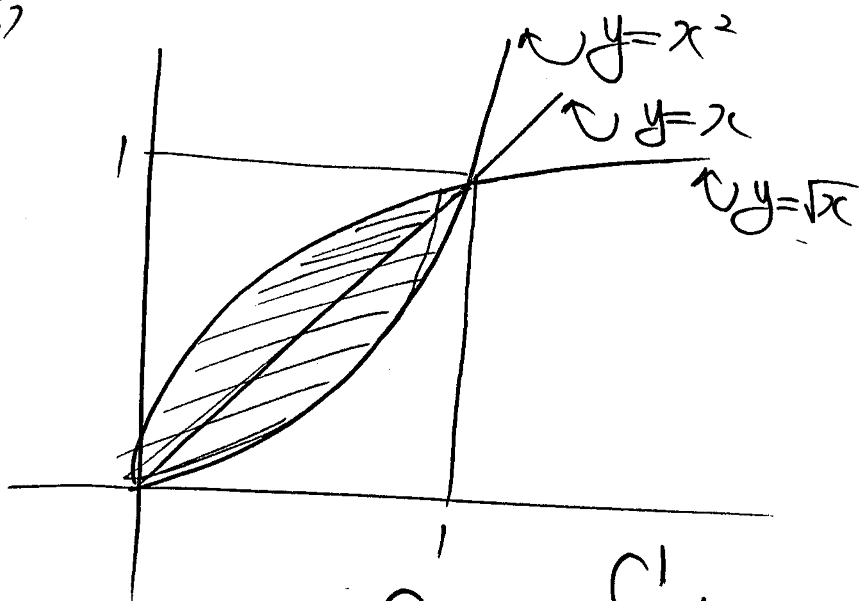


$$S = \int_a^b f(x)\,dx + \int_a^b (-g(x))\,dx$$

$$= \boxed{\int_a^b (f(x) - g(x))\,dx}$$

P313

EX7



$$S = 2\int_0^1 (x - x^2)\,dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3}$$

P315

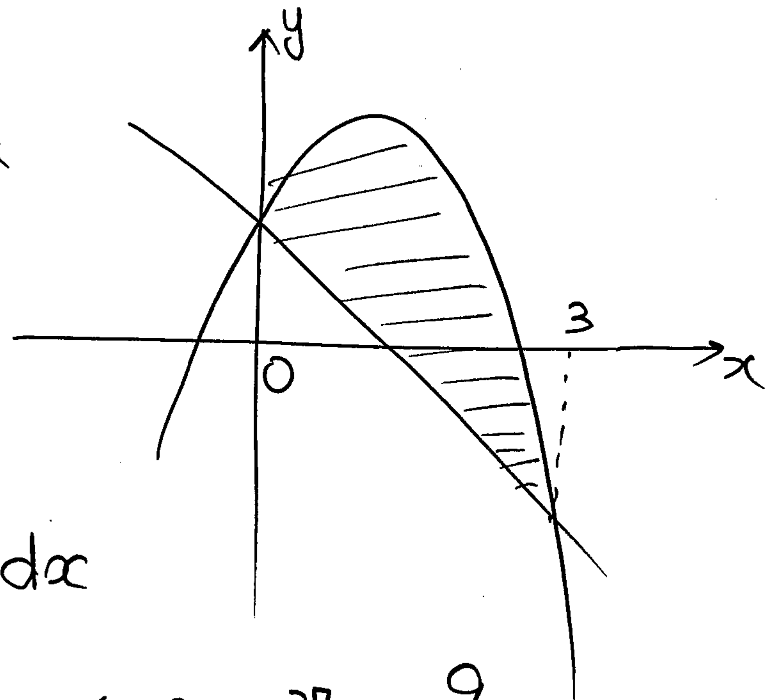
11 $\frac{9}{2}$

$-x^2+2x+1=1-x$

$x^2-3x=0$

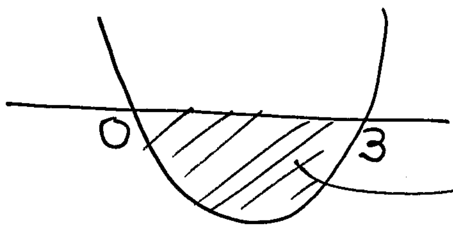
$x(x-3)=0$

$x=0$ 또는 $x=3$



$S=\int_0^3 (-x^2+3x)\,dx$

$=\left[-\frac{x^3}{3}+\frac{3x^2}{2}\right]_0^3=\left(-9+\frac{27}{2}\right)=\frac{9}{2}$



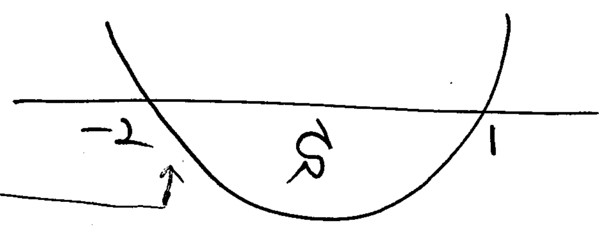
$S=\frac{(3-0)^3}{6}=\frac{9}{2}$

21

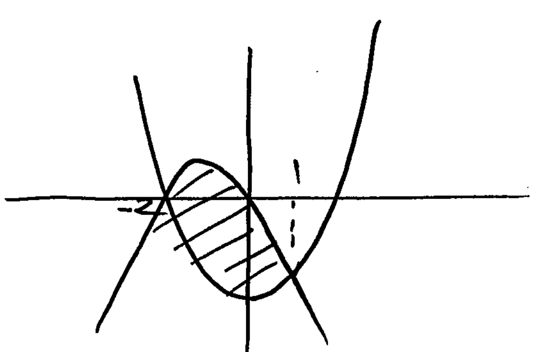
$x^2-4=-x^2-2x$

$2x^2+2x-4=0$

$\boxed{\begin{array}{c}2(x^2+x-2)=0\\ x=1 \text{ 또는 } x=-2\end{array}}$



$S=\frac{2\cdot(3)^3}{6}=9$



$S=\int_{-2}^{1}\{(-x^2-2x)-(x^2-4)\}\,dx$

$=\int_{-2}^{1}(-2x^2-2x+4)\,dx$

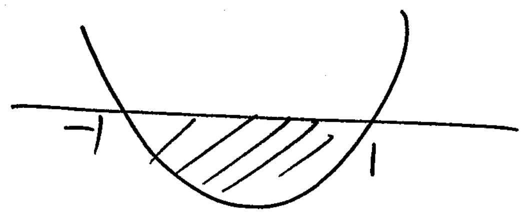
$=9$

3 | $\frac{8}{3}$

$-x^2+2x = x^2+2x-2$

$2(x^2-1)=0$

$x=\pm 1$



$S=\frac{2(2)^3}{6}=\frac{8}{3}$

P317

1-1 |

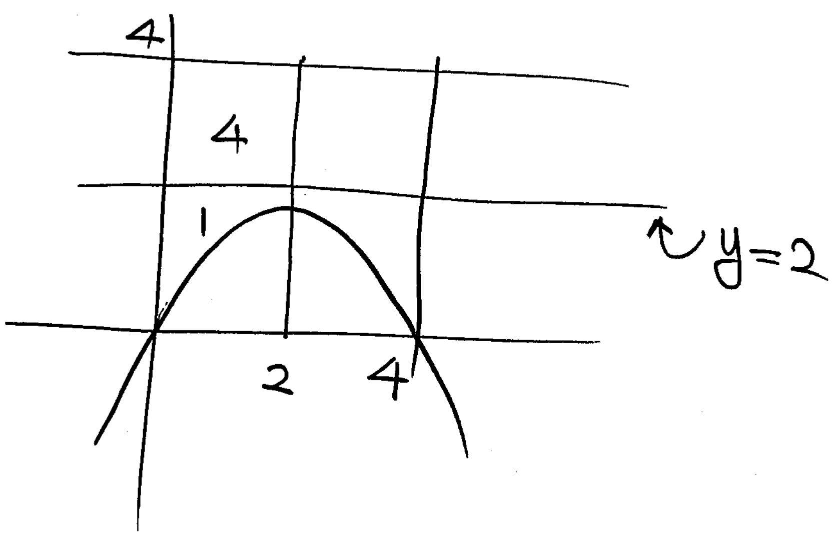
(1) $\left|\int_{-1}^{2} f(x)dx\right| = |S_1 - S_2| = \frac{27}{2}$

(2) $(\text{준식}) = 2\int_{-1}^{1} f(x)\,dx$

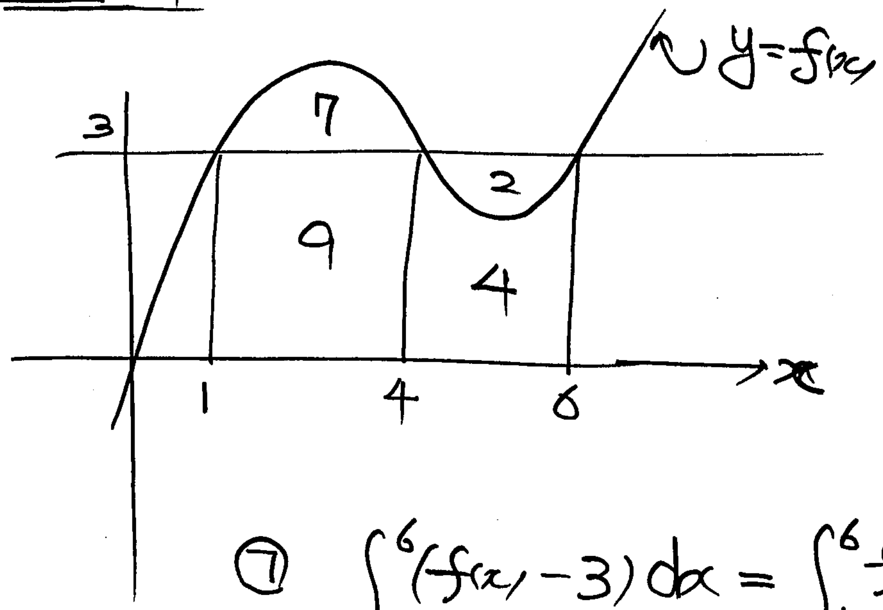
$= 2S_1 = 32$

$|f(x)|+f(x) = \begin{cases} 2f(x) & (-1 \le x < 1) \\ 0 & (1 \le x \le 2) \end{cases}$

1-2 | 1



1-3| 21



㉠ $\int_1^6 (f(x)-3)\,dx = \int_1^6 f(x)\,dx - \int_1^6 3\,dx$

$= 20 - 15 = 5$

㉡ $\int_3^6 f(x-2)\,dx$

$x-2=t$

$dx=dt$

$= \int_1^4 f(t)\,dt = 16$

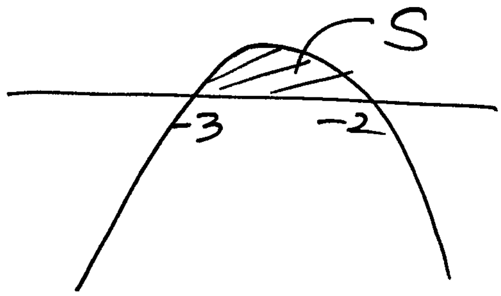
$\therefore\ 5+16=21$

2-11

(1) $-(x^2+5x+6)=0$

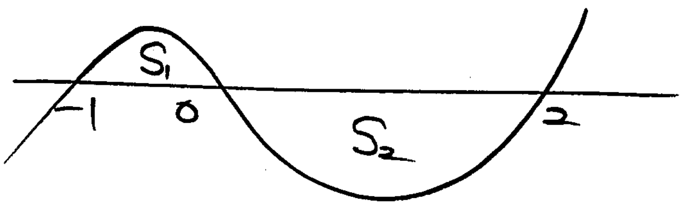
$x=-2$ 또는 $x=-3$

$S=\dfrac{1^3}{6}=\dfrac{1}{6}$



(2) $x^3-x^2-2x=0$

$x(x^2-x-2)=0 \qquad x=-1,\ 0,\ 2$



$S=S_1+S_2$

$S_1=\int_{-1}^{0}(x^3-x^2-2x)\,dx$

$=\left[\frac{x^4}{4}-\frac{x^3}{3}-x^2\right]_{-1}^{0}=0-\left(\frac{1}{4}+\frac{1}{3}-1\right)$

$=\frac{5}{12}$

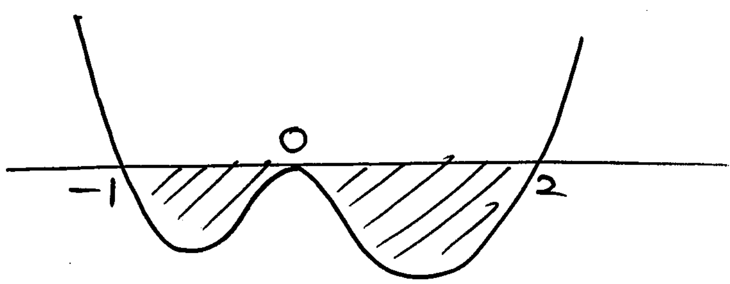
$S_2=\int_{0}^{2}(-x^3+x^2+2x)\,dx=\left[-\frac{x^4}{4}+\frac{x^3}{3}+x^2\right]_{0}^{2}$

$=-4+\frac{8}{3}+4=\frac{8}{3}$

$S=\frac{5}{12}+\frac{32}{12}=\frac{37}{12}$

(3) $x^4-x^3-2x^2=0$

$x^2(x^2-x-2)=0 \qquad x=0,\ 2,\ -1$



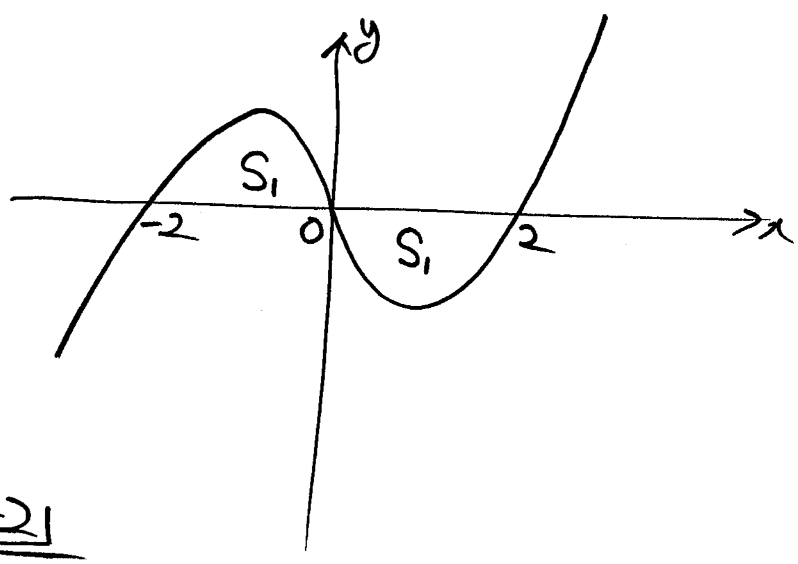
$S=\int_{-1}^{2}(-x^4+x^3+2x^2)\,dx$

$=\left[-\frac{x^5}{5}+\frac{x^4}{4}+\frac{2x^3}{3}\right]_{-1}^{2}$

$=-\frac{1}{5}(32-(-1))+\frac{1}{4}(16-1)+\frac{2}{3}(8-(-1))$

$=-\frac{33}{5}+\frac{15}{4}+6=\frac{-132+75+120}{20}=\frac{63}{20}$

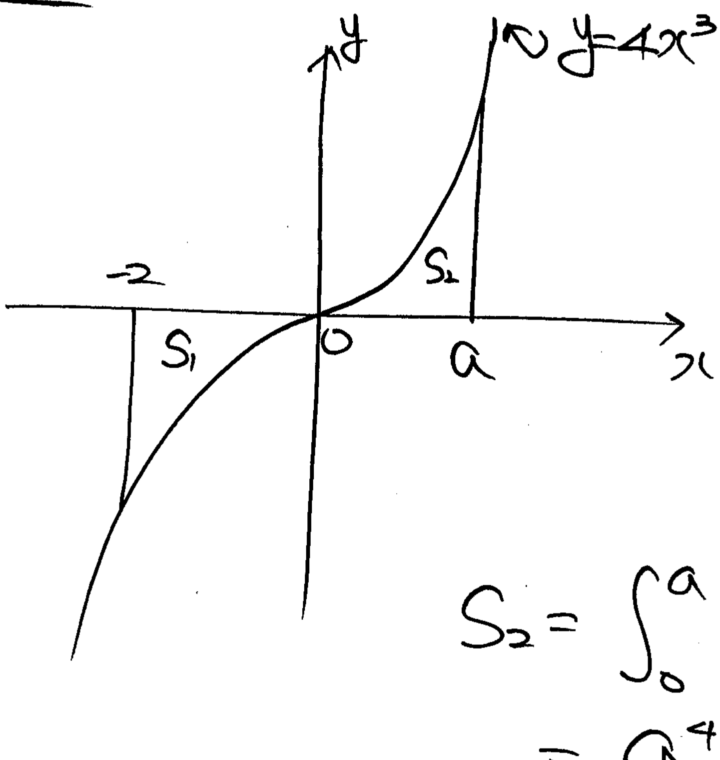
(4) $y = \begin{cases} x(x-2) & (x \geqq 0) \\ x(-x-2) & (x < 0) \\ = -x(x+2) & \end{cases}$



$S_1 = \frac{2^3}{6} = \frac{4}{3}$

$S = 2S_1 = \frac{8}{3}$

2-21



$S_1 + S_2 = 97$

$S_1 = \int_{-2}^{0} (-4x^3)\,dx$

$= \left[-x^4\right]_{-2}^{0}$

$= 0 - (-16) = 16$

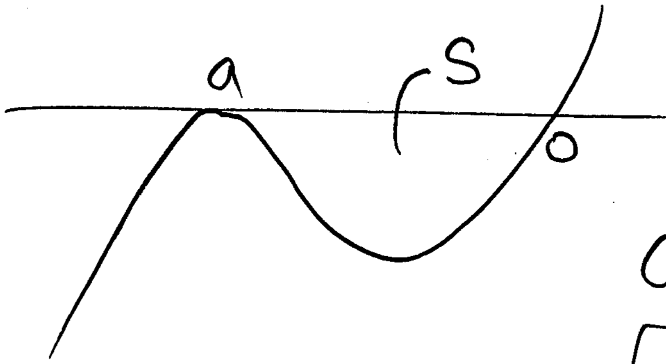
$S_2 = \int_0^a 4x^3\,dx = \left[x^4\right]_0^a$

$= a^4$

$16 + a^4 = 97$

$a^4 = 81 = 3^4 \quad \therefore a = 3$

(2)



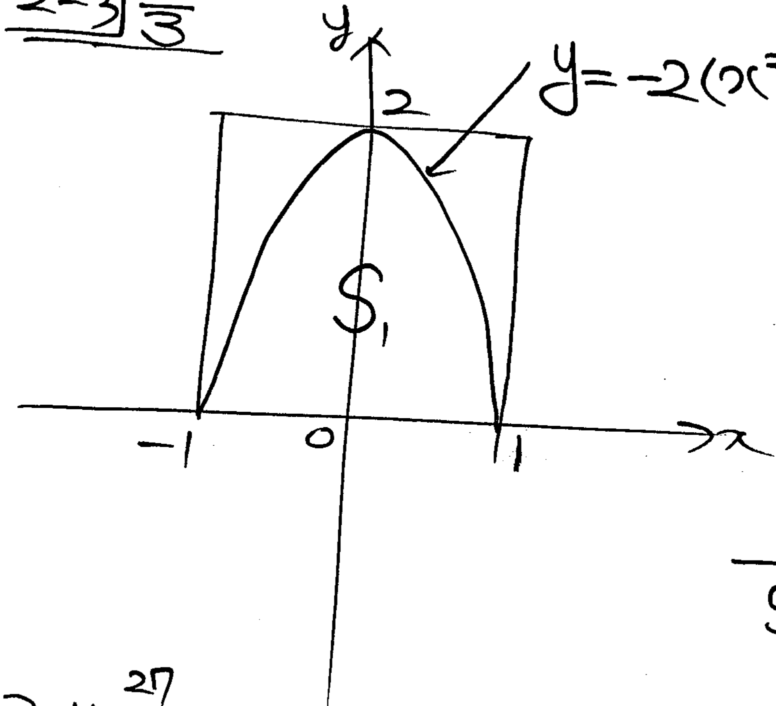
$$S = \frac{a^4}{12} = 108$$

$$a^4 = 36 \cdot 36 = 6^4$$

$$\boxed{a = -6}$$

2-3) $\frac{2}{3}$

$y = -2(x^2-1) = -2x^2+2$



$$S_1 = \frac{2 \cdot 2^3}{6} = \frac{8}{3}$$

$$S_2 = 4$$

$$\frac{S_1}{S_2} = \frac{\frac{8}{3}}{4} = \frac{8}{12} = \frac{2}{3}$$

3-1) $\frac{27}{4}$

$y = f(x) = x^3 - 3x^2 + x + 4$

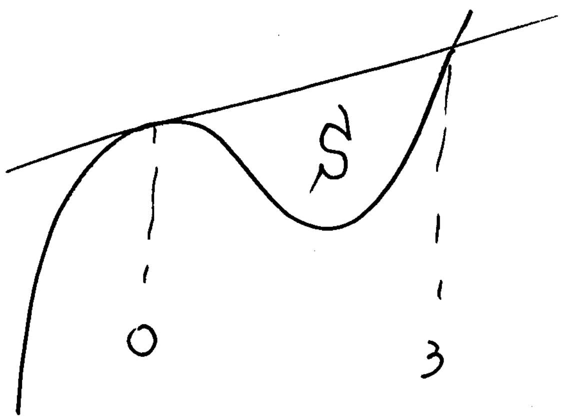
$f'(x) = 3x^2 - 6x + 1$

$f'(0) = 1$

접(0, 4) 기: 1

$y - 4 = 1 \cdot (x - 0)$

$y = x + 4$



$x^3 - 3x^2 + x + 4 = x + 4$

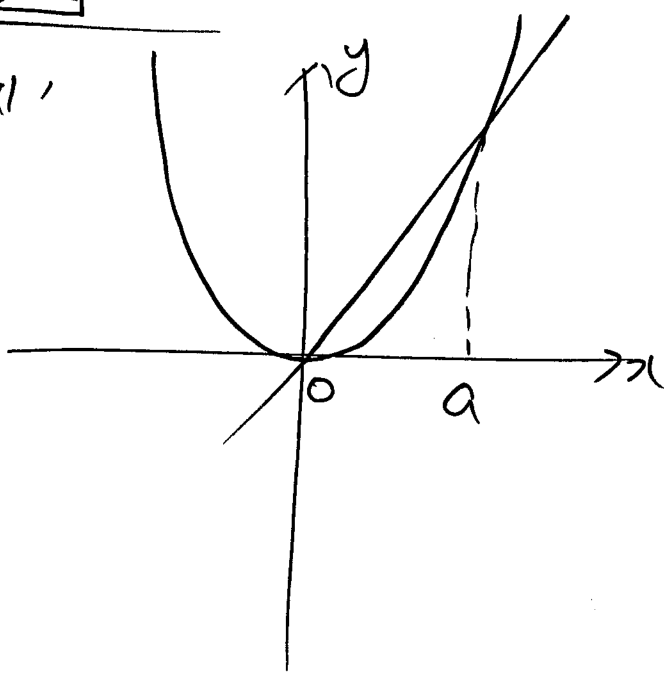
$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$$S = \frac{3^4}{12} = \frac{27}{4}$$

3-2)

(1)



$x^2 = ax$

$x = 0, a$

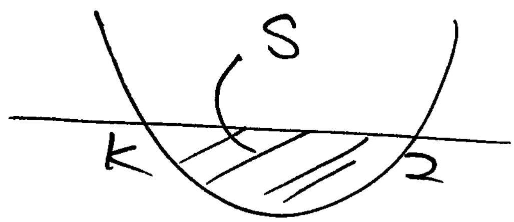
$S = \frac{a^3}{6} = 36$

$a^3 = 6^3 \quad a = 6$

(2) $x^2 + 2k = (k+2)x$

$x^2 - (k+2)x + 2k = 0$

$(x-2)(x-k) = 0$



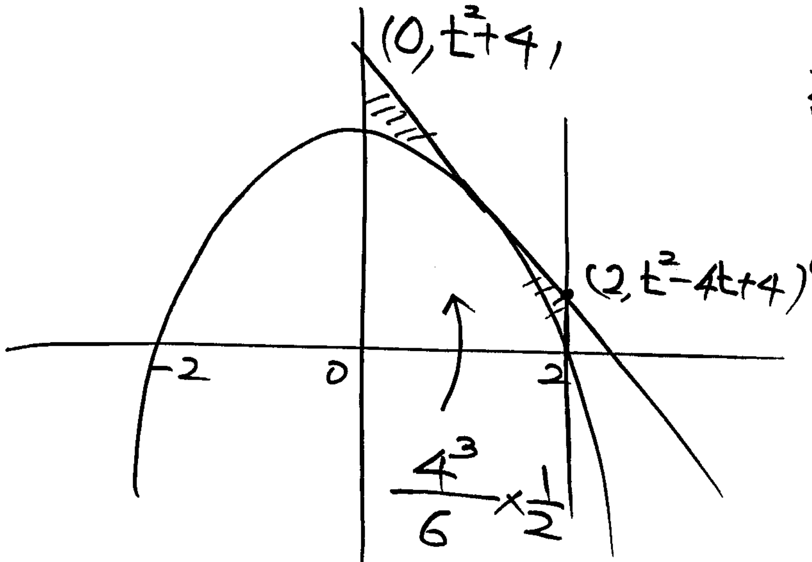
$S = \frac{(2-k)^3}{6} = 36$

$(2-k)^3 = 216 = 6^3$

$2 - k = 6$

$\boxed{k = -4}$

3-31 $\frac{2}{3}$



$\frac{4^3}{6} \times \frac{1}{2}$

$= \frac{16}{3}$

$y' = -2x$

점 $(t, 4-t^2)$ 기: $-2t$

$y-(4-t^2) = -2t(x-t)$

$y = -2tx + t^2 + 4$

$S(t) = \cancel{\frac{1}{2}} \times (2t^2-4t+8) \times \cancel{2} - \frac{16}{3}$

$= 2t^2 - 4t + \frac{8}{3}$ $\quad t=1;\ m = S(1) = \frac{2}{3}$

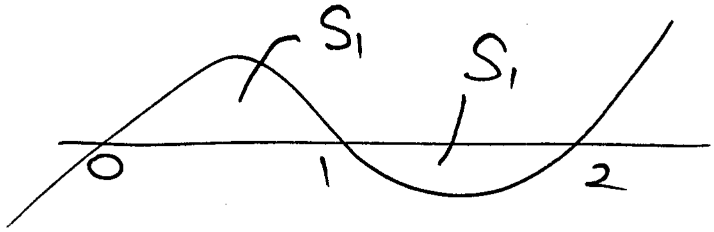
$= 2(t-1)^2 + \frac{2}{3}$

4-11

$x^3 - 2x^2 = x^2 - 2x$

$x^3 - 3x^2 + 2x = 0$

$x(x-1)(x-2) = 0$



$S_1 = \int_0^1 (x^3 - 3x^2 + 2x)\,dx = \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1$

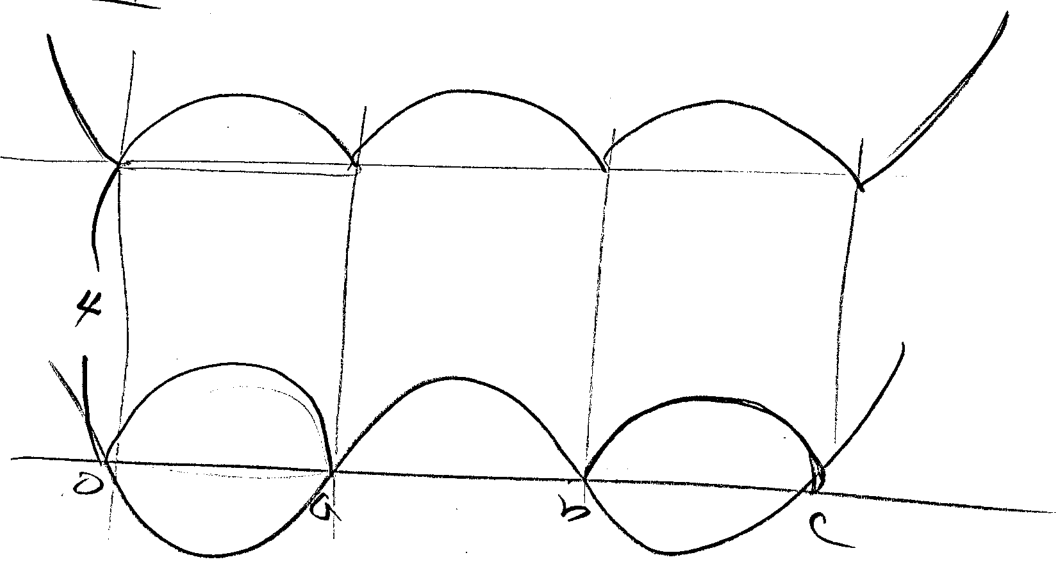
$= \frac{1}{4} - 1 + 1 = \frac{1}{4}$

$S = 2S_1 = \frac{1}{2}$

4-2) $\frac{27}{4}$

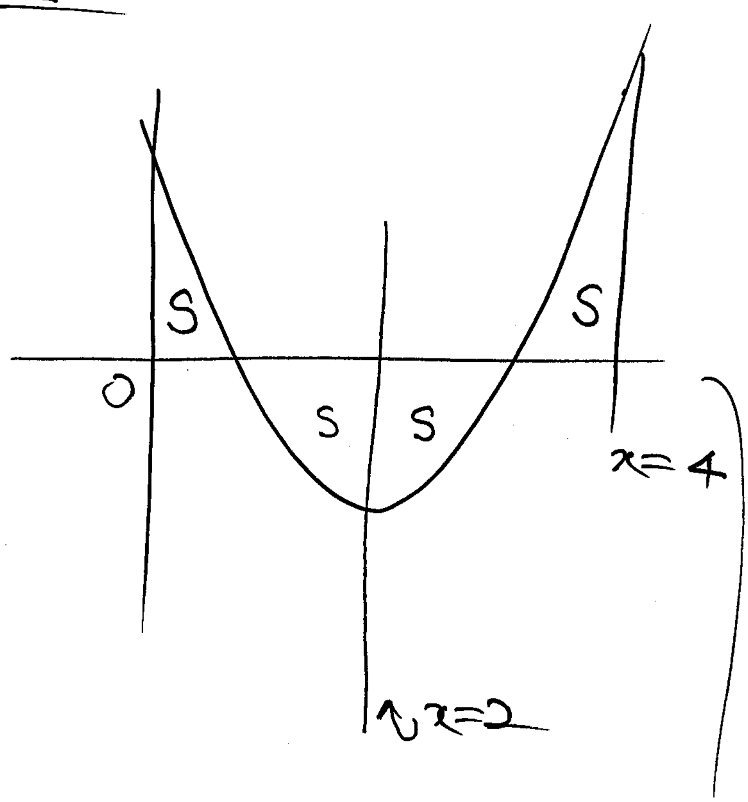
$$S = \frac{3^4}{12} = \frac{27}{4}$$

4-3) 24



$$S = 6 \times 4 = 24$$

5-1)



$$\int_0^2 (x^2 - 4x + k)\,dx$$

$$= \left[\frac{x^3}{3} - 2x^2 + kx\right]_0^2$$

$$= \frac{8}{3} - 8 + 2k$$

$$= 2k - \frac{16}{3} = 0$$

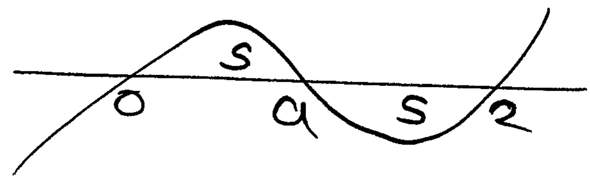
$$k = \frac{8}{3}$$

5-2)

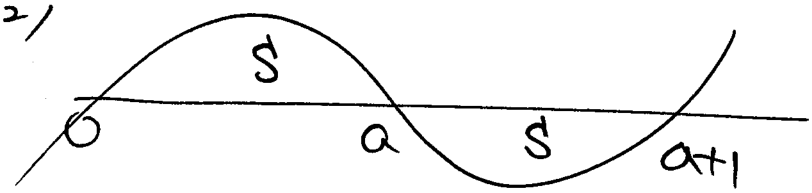
(1) $y = x^3 - (a+2)x^2 + 2ax$

$= x(x-a)(x-2)$

$a = 1$

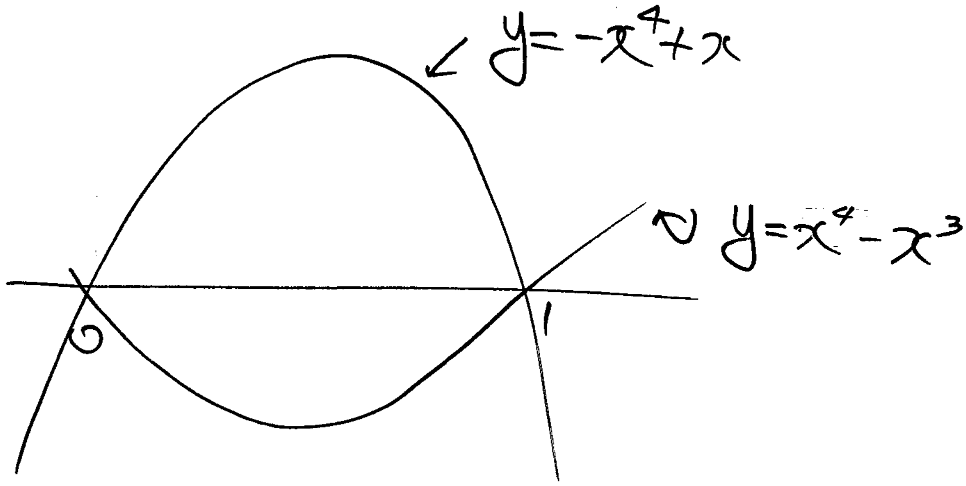


(2)



$a = 1$

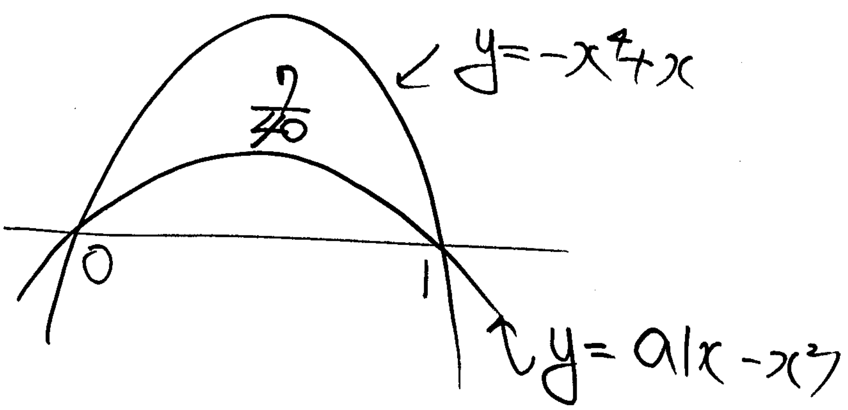
5-3) $\frac{3}{4}$



$$S = \int_0^1 (-2x^4 + x^3 + x)\,dx$$

$$= \left[-\frac{2x^5}{5} + \frac{x^4}{4} + \frac{x^2}{2}\right]_0^1 = -\frac{2}{5} + \frac{1}{4} + \frac{1}{2}$$

$$= \frac{-8+5+10}{20} = \frac{7}{20}$$



$$\int_0^1 \left(-x^4 + x - a(x - x^2)\right) dx$$

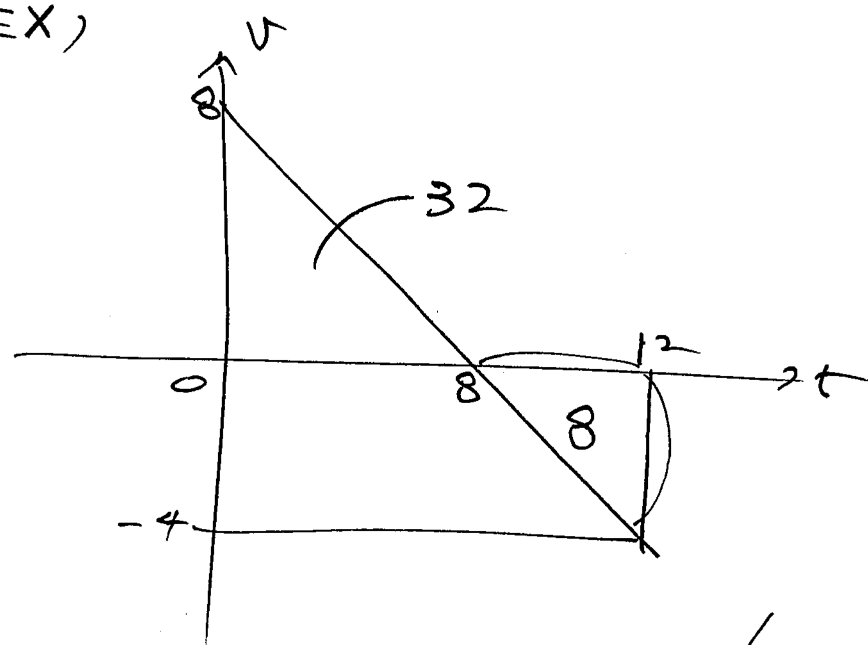
$$= \left[-\frac{x^5}{5} + \frac{x^2}{2} - a\left[\frac{x^2}{2} - \frac{x^3}{3}\right]\right]_0^1$$

$$= -\frac{1}{5} + \frac{1}{2} - a\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{3}{10} - \frac{a}{6} = \frac{7}{40}$$

$$\frac{a}{6} = \frac{12-7}{40} = \frac{1}{8} \qquad \boxed{a = \frac{3}{4}}$$

P327

EX)

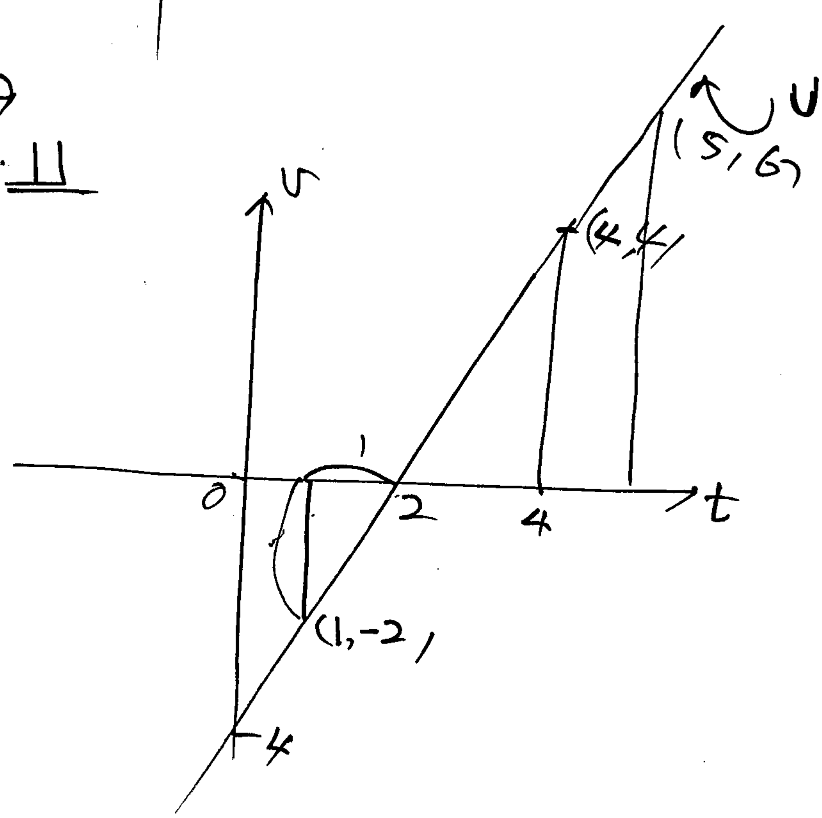


$t = 8$; 32

$t = 12$; 24

P329

II

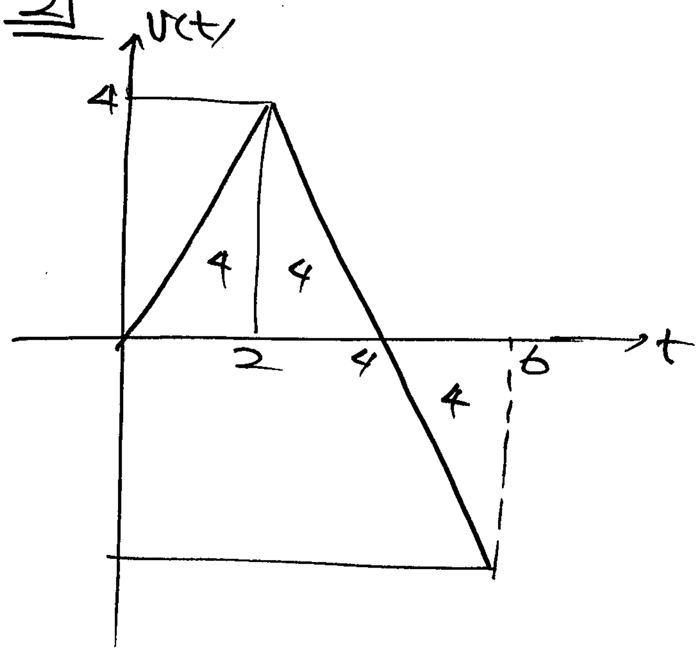


$v(t) = 2t - 4$

1) 0

2) 8

21

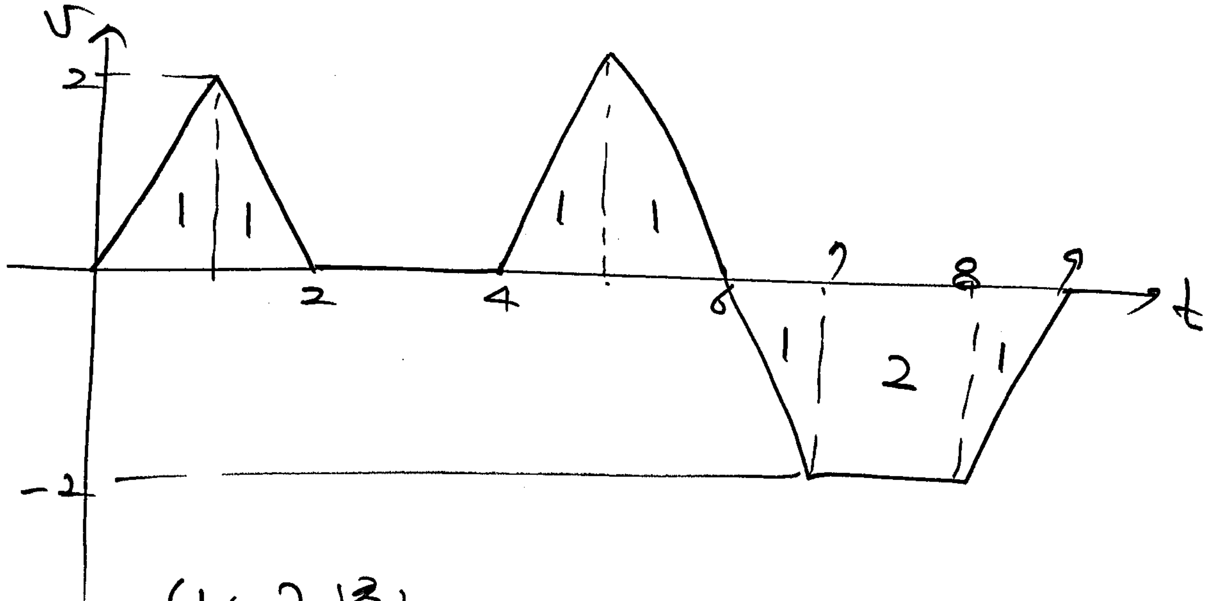


(1) 0

(2) 8

P33 1

6-1

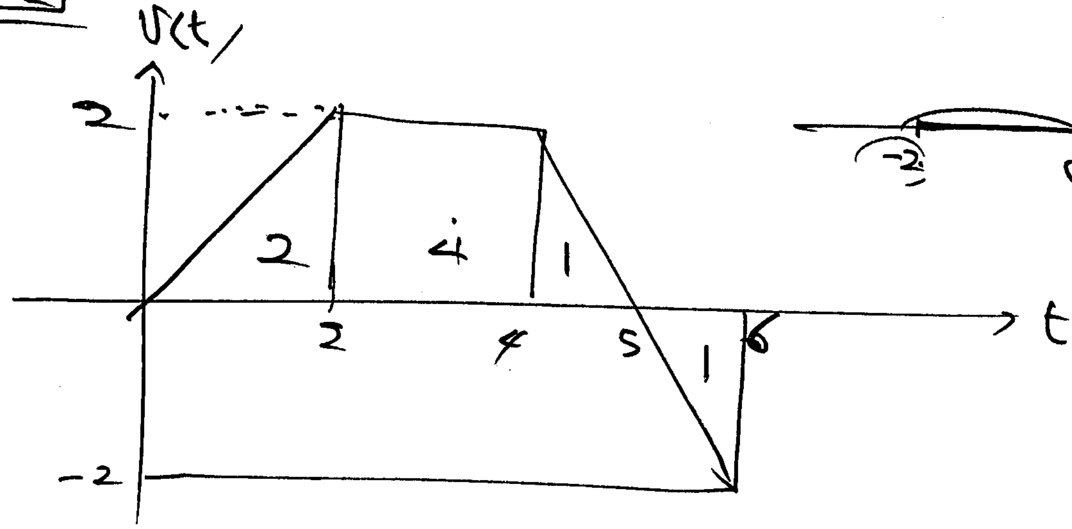


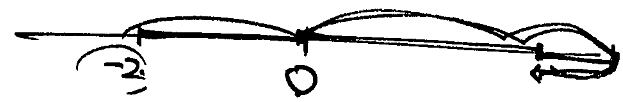
(1) 2 초

(2) 9 초

(3) 8

6-2)



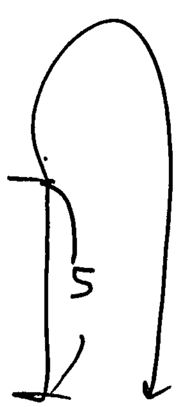


ㄱ ㄴ ~~ㄷ~~

6-3)

ㄱ ~~ㄴ~~ ㄷ

7-1)

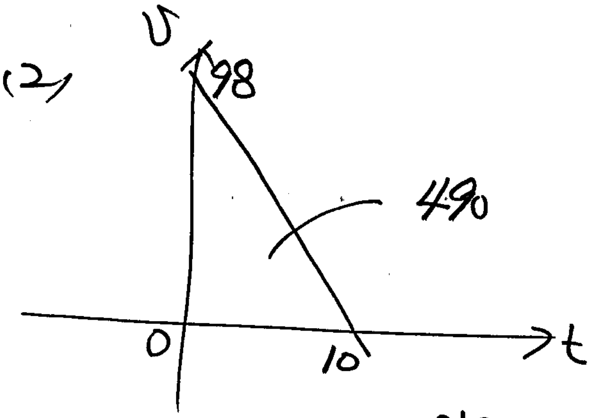


(1) $h = 5 + \int_0^1 (98 - 9.8t)\,dt$

$= 5 + [98t - 4.9t^2]_0^1$

$= 5 + 98 - 4.9 = 98.1$(m)

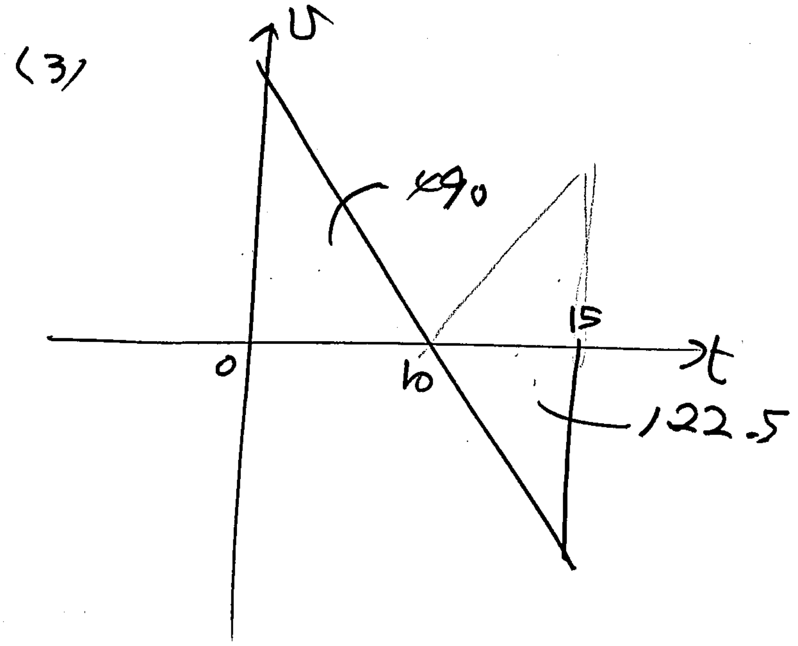
(2)



495(m)

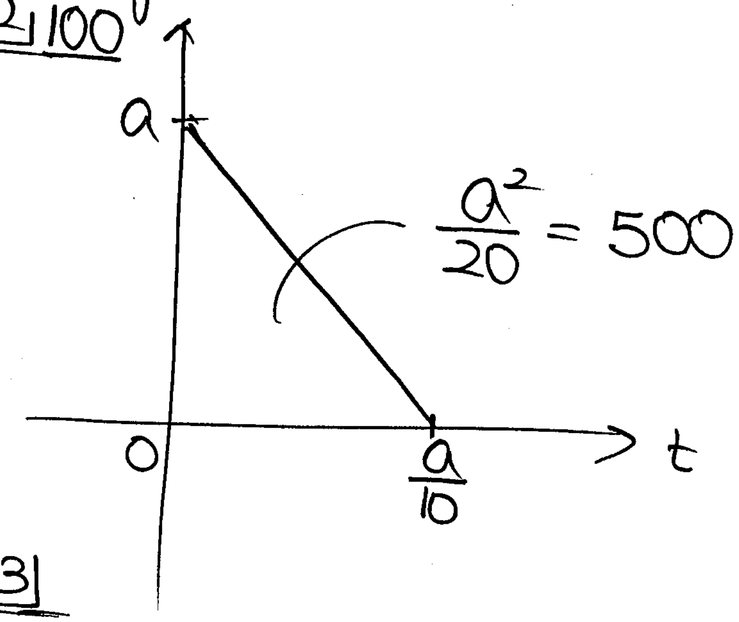
$h = 5 + \int_0^{10} (98 - 9.8t)\,dt$

(3)



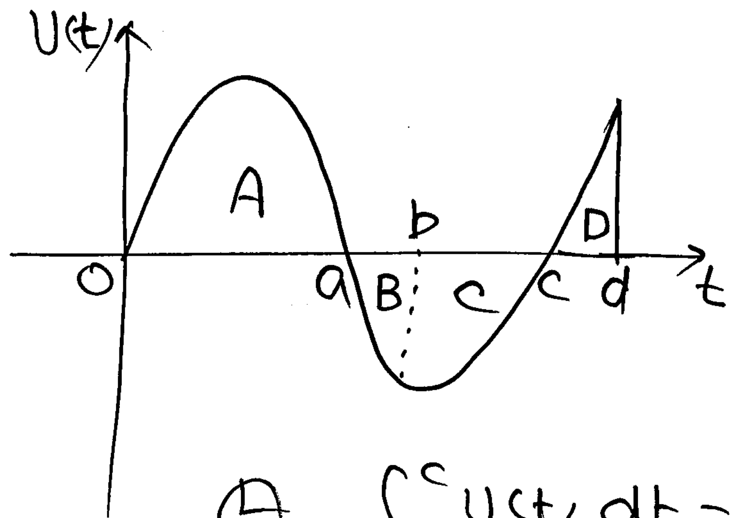
$490 + 122.5$
$= 612.5$

7-2 100



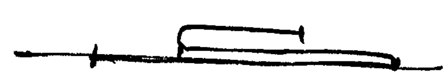
$a - 10t = 0$
$t = \frac{a}{10}$
$a^2 = 10000$
$a = 100$

7-3



$A = B + C + D$

$\neq$



4 $\int_0^c v(t)\,dt = A - B - C$

$\int_c^d v(t)\,dt = D$

2 $\int_0^b v(t)\,dt = A - B$ $\quad \int_b^d |v(t)|\,dt = C + D$